

# The AQUA Data Model and Algebra\*

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## Abstract

This paper describes a new object-oriented model and query algebra that will be used as an input language for the query optimizers that are being built as a part of the EREQ project. The model adopts a uniform view of objects and values and separates syntactic, semantic, and implementation concerns. The algebra addresses issues of type-defined equality and duplicate elimination as well as extensions to bulk types other than sets.

## 1 Introduction

Recently, a great deal of work has been done on the topic of object-oriented query algebras [29, 24, 12] and the modeling of bulk types [4, 27, 21]. These proposals, as well as those of other researchers on the topic, have explored some of the fundamental issues and provided the starting point for the work reported here. AQUA (A QUery Algebra) is the result of a joint effort among researchers who have participated in the design of previous algebras [28, 32, 33].

AQUA has been designed to address a number of detailed modeling issues that we believe needed further work, but the overarching goal for this work has been the design of an algebra that would serve as the input to a broad class of query optimizers. In this way, it could be used as a de facto standard in the construction of object-oriented query optimizers. It would serve as the target language for user-level query languages. Any user language for which there was a translator to AQUA could then be processed by any of the optimizers that are designed for AQUA. Thus, AQUA is an intermediate language between the user's query and the query optimizer.

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AQUA and the data model on which it is based are strongly typed and are designed to deal correctly and uniformly with abstract types. It should be kept in mind that although we often talk about tuples in this context, they are abstractions as well and are not the stored representation of the objects.

AQUA is closed in the sense that all of its operators return objects that are defined in the model. It should also be pointed out that although AQUA supports updates through the use of methods, we do not discuss this here.

Any query language or algebra must be embedded in some data model. We have attempted to provide a simple model that would be general enough to cover the modeling concepts in other object-oriented models. We have adopted a model in which everything is an object in the sense that it has a well-defined interface and can be referenced from other objects, yet, we also support value-based semantics as described later in this paper.

AQUA has been designed to be very general. We have purposely included a rich set of operators that could directly simulate the operators in other proposals. AQUA is not a minimal set of operators. Redundancy allows us to more easily accommodate many query languages, and, at the same time, allows us to have a powerful and varied set of optimization strategies. Operator redundancy supports optimization by making query rewrites possible.

Another goal of our work has been to support many different bulk types in a uniform manner. We have designed AQUA in such a way that it will not preclude additional bulk types like lists or arrays. The AQUA design also addresses the problems introduced by type-specific equalities by providing special functions that deal with equivalence classes and duplicate elimination in a bulk type object based on some equality specific to the element type. The presentation in this paper covers two bulk types, sets and multisets. The discussion of more complex, ordered types is beyond the scope of this paper [31].

Some of the operators that one would expect to find in any algebra appear in AQUA as well. They have, however, been generalized to deal with many data models and many possible bulk types. For example, our **join** operator (see Table 4) takes the standard three arguments - the two input sets or multisets and a matching predicate. It also takes an extra argument which is a function of two inputs that combines every pair of objects from the two input sets that satisfy the matching predicate. This can cover **joins** that produce sets of pairs or **joins** that behave like **semi-joins**, for example. If the two input sets were sets of lists, it could combine the two elements by concatenation, thereby producing a set of lists.

AQUA is currently being used as the input language for two prototype extensible optimizers, Epoq at Brown University [23] and Revelation at OGI [11]. It appears to give us the power that we need for expressing complex queries while, at the same time, it seems to cover the functionality of all the query algebras of which we are aware.

## 2 Related Work

One of the primary goals of AQUA is to provide a model general enough to simulate the constructs of any object-oriented data model (and most value-oriented models), no matter what choices it makes with respect to certain features (bulk

types, encapsulation, identity versus value, notions of equality, inheritance, and operations (up to a point)). Other models have claimed similar goals, but not necessarily in all these areas at once, and our mechanisms for achieving these goals differ substantially from those of our predecessors. Many of the specific constructs of AQUA were inspired by or drawn from the EXTRA/EXCESS system [33], ENCORE/EQUAL [28], and Revelation [32].

AQUA is intended to support large numbers of bulk types and to do so in a flexible, uniform way, such that the addition of other bulk types later on will be straightforward. [6] proposes a meta-level algebra for collections of complex objects with identity and also includes some transformation rules for optimization. This algebra, however, does not correspond to a specific data model but rather to a higher-level notion of collections of objects. Its operations and rules are templates that are intended to be “instantiated” in actual systems according to certain parameters of the specific data model being implemented. Thus it takes a different approach to generality than does AQUA. It also does not support several of the constructs of AQUA (including grouping and immutable semantics). EXTRA/EXCESS [33] also attempts to support a large number of bulk types, but does not explicitly provide sets (they are provided only by eliminating duplicates from multisets). The inclusion of a union type is not new (see [19]), but we provide it with a clean algebraic interface using both **tagcase** and **typecase** constructs to be fully general. The importance of being flexible about the addition of new bulk types has been established (see [21]); the modularity of the AQUA approach facilitates this to an extent by following a rationale similar to that of Rozen and Shasha (see [27]) in several respects.

In attempting to support both values and objects, some systems (e.g. EXCESS [33]) choose to support only values in the type system and to model objects by using explicit identifiers. Other systems (e.g. Smalltalk and ORION [13, 5]) choose to support only objects in the type system and to model values as a special case of objects. IQL (see [1]) defines two separate languages, one enforcing object identity and one not supporting it at all. Our characterization of the distinction between “objects” and “values” as the difference between entities (objects) with mutable and immutable semantics provides a much cleaner formalism, and was partially inspired by systems such as Larch [14]. By cleanly separating the notions of type (a syntactic concept) and semantics we provide a model that treats both values and objects as first-class citizens and has a simpler type system. We are not aware of another model that takes this approach, nor of one that takes the clearly-separated, 3-level view of an object that we do (type, semantics, and implementation; see Section 3.1). Buneman and Ohori (see [8]) exhibit a similar philosophy, though, in their distinction between a *kind* and a *type*.

C++ [30] has a notion of “const” that is similar to our notion of “immutable”, but in C++ this notion is part of the type system, and thus causes a variety of problems that motivated us to separate type and semantics. Eiffel [22] makes a distinction between reference and copy semantics, but not between mutable and immutable semantics. Unlike ILOG (see [15]) and others, we avoid explicit identifiers in the model, viewing them as an implementation concern, and reflecting the distinctions between objects and values by using varying semantics (see Section 3.3).

It has been pointed out by Atkinson et al. (see [4]) that in object-oriented

systems, a type may supply its own method for testing equality. This capability, however, introduces problems such as what is the meaning of operators like set union that depend on equality for their own semantics. The AQUA approach to enforcing a notion of equality among the elements of a set seems to be without precedent because in many cases there is no need for any notion of equality other than object identity but it still allows the (non-deterministic) creation of sets whose members are determined by an arbitrary equivalence relation. Many models (e.g. MDM [25]) do not have this flexibility.

Most “pure” object-oriented models ([13, 20], and others) provide and enforce encapsulation of data types. In AQUA our notion of type is more general: not everything is forced to be of an encapsulated, abstract data type whose only interface is that provided by the definer of the type. But AQUA does support such types, and does so using the “abstraction” type constructor, allowing any database object to be described using a single uniform type system. This is similar to the ADT concept provided by Postgres [26], but more general in the sense that any type definable in the AQUA type system can be abstracted into a true encapsulated type, and an abstraction in AQUA is a first-class citizen of the type system – the abstraction constructor has the same status as any other constructor. The distinction is that an object of an abstraction type has only the user-defined methods as an interface, while an object of (for example) a set type has an interface consisting of union, difference, etc. This is similar to Postgres’s notion of user-defined Postquel functions and the functions and procedures of EXCESS [9], but in those systems, the ability to define functions allows one to add operations to an existing, non-encapsulated type (i.e., encapsulation is not enforced in those systems but is in AQUA). AQUA can, of course, emulate these features of Postgres and EXCESS. The use of a type constructor to represent abstraction enables all objects in an AQUA database to exist in one seamless type system. Our approach is similar to that of [3], but in their model not everything is an object, so their equivalent to our abstraction constructor must enforce many more of the facets of “objectness” than must ours.

Several of our operators resemble operators of ENCORE/EQUAL, EXTRA/EXCESS, and Revelation. See Section 5 for detailed descriptions of the operators. **Fold** comes directly from Revelation, but we have adapted it for use with both sets and multisets, as it is basically a form of structural recursion ([7]). **Apply** is similar to mapping-style operators of other models; it is closely related to the SET\_APPLY operation of EXCESS. Our **nest** and **unnest** operations are generalizations of those defined in ENCORE/EQUAL, and the **group** and **set** operations are also found in EXCESS. **Dup\_elim** for both sets and multisets (as we’ve defined it) and **convert** appear to be original to this model, with **dup\_elim** being by far the more interesting. AQUA’s **dup\_elim** can be thought of as a generalization of other duplicate elimination operators (e.g. that of ENCORE/EQUAL [28]). Our binary **join** operation is similar to the n-ary **image** operator of MDM [25], but differs from it in that we separate the join predicate from the function to be applied to matching pairs; the idea of this is to enhance optimization by making certain queries (e.g. equijoins) easier to recognize. A set-theoretic **choose** operator appears in the algebras of Osborne and MDM (see [24, 25]). Non-determinism is also present in [2], which describes a *witness* operator which operates in a logical (rather than an algebraic) setting and creates a set of possible interpretations of a formula, resulting

in non-determinism. Also, the decision to make the boolean operators (**and**, **or**, and **not**) full-fledged algebra operators, rather than constructs available only in certain parts of the language (e.g. predicates), as in the relational algebra, EXCESS, and Straube's algebra (see [33, 29]), adds to the flexibility of the algebra. Finally, the type parameter to **union**, **difference**, and **intersection** is similar to that used in EXCESS.

### 3 The AQUA Data Model

The AQUA data model is founded on the notions of strong typing and abstract data types. In the AQUA data model, an object has a type, and its state is accessed and modified via a well defined set of interface functions. The *state* of an object is a mapping from AQUA objects to *mathematical values*. All AQUA objects are unique, and this uniqueness can be detected by the user. AQUA objects have identity, and can be distinguished using equality that is based on identity. We do not specify the implementation of identity, to prevent a fixation on object identifiers. The most important point is that objects are unique, and that given two objects, we can determine whether or not they are the same object.

Everything that is stored in an AQUA database is an object. Database objects can have global names which facilitate access to the corresponding object. This approach allows the model to have a uniform flavor. The expressive power of the model is not crippled by this uniformity, as we shall see below.

The uniformity of the data model allows us to easily and consistently define algebraic operators over collections of objects and values.

#### 3.1 A Three Tiered Object Model

A common view of types is that they are a description of various aspects of the meaning or behavior of that type. In the AQUA type system, we have separated the notion of "syntactic" type from the notion of "semantic" type. Types are a syntactic property of names. Each type has an associated non-empty set of semantics, each of which provides a description of meaning and behavior that objects of the type can have. Every type provides a set of interface functions, since everything in AQUA is an object. The interface presented by the type is a set of signatures for the interface functions. The meaning and implementation of the interface functions is determined by the semantics and implementation of a given instance of the type.

An AQUA *type* is defined recursively:

$\mathcal{B} = \{integer, float, boolean, string\}$   
(the set of base types)

(name, hier,  $C(m_1 : t_1, \dots, m_n : t_n)$ )

$m_i$  is a name,  $t_1, \dots, t_n$  are types or objects, and  $C$  is a type constructor.

The *hier* symbol represents the set of immediate supertypes of the type being defined. The data model supports subtyping via the notion of substitutability. Since our types are syntactic, substitutability is also syntactic. We use the symbols  $\sqsubseteq$  and  $\sqsupseteq$  to indicate subtype and supertype relationships between

types. Functions have types, although our notation (described below) only allows the instantiation of particular functions. The name equivalence rule is used for determining the equality of two AQUA types.

The *semantics* of a type might loosely be thought of as a Larch [14] specification, which axiomatically describes properties of the operations on a type. The particular language used for describing semantics is a topic of our current research.

Specifying semantics separately from types allows different instances of the same type to have different behaviors. For example, making this distinction aids us in our goal of viewing all entities as abstract data types by moving the traditional distinction between objects and values into the semantics layer. This is discussed in more detail below. At present, we define two possible semantics, mutable and immutable. Objects with mutable semantics may update their state, while objects with immutable semantics may not.

Another example of the use of semantics is the declaration that certain operations are commutative. A query optimizer could then make use of such information when optimizing a query. The semantics describing commutativity for deques might be written as  $notEmpty(q) \Rightarrow enqueue(dequeue(q), x) = dequeue(enqueue(q, x))$ . Commutativity axioms for sets might include:  $select(p2)(select(p1)(s)) = select(p1)(select(p2)(s))$  or  $join(p, f)(A, B) = join(p, f)(B, A)$ , provided  $p$  and  $f$  are commutative. See Section 5.1 for a complete description of the AQUA **join** operator).

Each semantics of a type may have multiple *implementations*, resulting in the third tier of our three tiered type system. An implementation is a description of how an instance of a type with a particular semantics is to be implemented (in terms of data structures and algorithms). The specification of implementation allows the database or a user to select the most desirable implementation at any point in the lifetime of the object. Thus, an object really is an instance of a particular implementation of a particular semantics of a type. A result of our approach is that type can be determined statically (at compile-time), but semantics and implementation can only be determined at run time.

### 3.2 Constructing Types

A type constructor is a meta type which defines a family of types. The *Set* type constructor defines the family of types that includes *Set[Person]*, *Set[Department]*, etc. New types are created by instantiating the meta type with a specific type, like *Person* or *Department*.

The interface of a new type is determined by the particular type constructor and its parameters. AQUA types with user-defined interfaces may also be defined using a special “*abstraction*” type constructor, defined below. All types defined using the *abstraction* constructor must provide their own **new** operation for creating objects of that type. The *operators* of the AQUA algebra are the methods on the AQUA type constructors.

We define the following standard type constructors; *Set[T]* – a collection of unique entities of the same type; *Multiset[T]* – a bag of entities of the same type (i.e. an entity may appear more than once in a multiset); *Tuple[l<sub>1</sub> : T<sub>1</sub>, . . . , l<sub>n</sub> : T<sub>n</sub>]* – a fixed-length list of labeled entities of (possibly) different types; *Union[l<sub>1</sub> : T<sub>1</sub>, . . . , l<sub>n</sub> : T<sub>n</sub>]* – a tagged union, similar to that found in

many programming languages;  $Function[T_1, \dots, T_n, T_r]$  – a named list of types, the last of which is the output type of the function (the others being the input types);  $Abs[f_1 : T_{f_1}, \dots, f_n : T_{f_n}]$  – abstraction. An abstraction is a new abstract data type and has a collection of named functions, which form the interface of the abstraction. (We leave the representation of the abstraction to the implementation and do not discuss it here);  $N\text{-dimensional array}[T](n, l)$  – an n-dimensional collection of entities of the same type; individual entities can be accessed directly. The parameter  $l$  is a list of  $n$  dimensions; at most one of the dimensions may be of variable length;  $List[T]$  – a sequence of entities of the same type;  $Tree[T]$  – a tree whose nodes all contain entities of the same specified type; and  $Graph[T]$  – a graph whose nodes all contain entities of the same specified type. Lists, trees, and graphs are not discussed further here; see [31] for a complete description of them. N-dimensional arrays are a subject of our future research.

*Abstractions, tuples and unions* are subtypeable. Functions are subtypeable using the standard contravariance rule. *Sets* and *multisets* are not subtypeable. As a reminder, the principle of substitutability states that a subtype may be used anywhere its supertype may be used. If we allowed sets to be subtypeable, then given a variable  $zoo$  with type  $Set[Animal]$ , and  $Dog \sqsubseteq Animal$ , then we could assign  $zoo$  an object of type  $Set[Dog]$ , and then we could also incorrectly insert the elephant Dumbo, an animal, into  $zoo$ , a  $Set[Dog]$ . A similar argument shows that multisets are not subtypeable.

### 3.3 Our Approach to Values

Mutable and immutable semantics are the key to incorporating the traditional notions of object and value into the AQUA model. In the discussion that follows, we use object to refer to the traditional notion of object, and use AQUA object to refer to objects as they appear in AQUA. There are (so far) two possible semantics for AQUA objects, mutable and immutable. Other semantics can be defined, and AQUA objects may have any semantics that has been defined in the system and is applicable to that type.

AQUA supports the traditional notion of values via immutable semantics for objects. If an AQUA object is immutable, its contents can never change, and it becomes impossible to detect whether or not it is being shared — it takes on the role of a value. The AQUA base types only support immutable semantics. Traditionally, objects are used to achieve sharing, and values are used to prevent sharing.

The type system will allow an immutable AQUA object to be assigned to a variable referencing a mutable AQUA object. Variables only have a type, not a semantics. For example, semantics allow us to create a single type, say *Person*, which has mutable and immutable semantics. We can then create a mutable *Person* and an immutable *Person*, and use either one where a *Person* is required. The behavior of the program depends on whether or not the *Person* is mutable or immutable at the time that the code is executed. As discussed in the next section, this allows us to have type compatible *Person* “objects” and *Person* “values”.

### 3.4 Examples

We will use the following schema definitions to show the use of the abstraction type constructor, and to set the stage for some sample queries described later. The examples use the abstraction constructor to specify the interface of the abstract data type, and show a name implementation specification to describe the representation of the abstract type. A *Company* is an abstract data type which uses a tuple as its representation, and which supplies a set of methods to access and update its name and address. *Person* is similar, but it also has an Employer field which is of type *Company*. We also have a set of *Company* objects,  $Companies = Set[Company]$  and a set of *Person* objects,  $Persons = Set[Person]$ . No semantics is presented since our language for defining semantics is incomplete.

```
type Company =  
  abs(name(Company) -> String;  
       change_name(Company, String);  
       address(Company) -> String;  
       change_address(Company, String);  
  )
```

```
Implementation CompanyA for Company =  
  Tuple[name : string, address : string]
```

```
type Person =  
  abs(name(Person) -> String;  
       address(Person) -> String;  
       employer(Person) -> Company;  
  )
```

```
Implementation PersonA for Person =  
  Tuple[name : string, address : string,  
        employer : Company]
```

## 4 The AQUA Approach to Algebra Design

In this section, we discuss some of the design issues related to the AQUA algebra, including the syntax of terms, the use of type parameters, and the treatment of equality.

### 4.1 Syntax

Expressions in the algebra are represented by *terms*. A term is either: a variable, constant or function symbol, a *lambda abstraction* of the form  $\lambda(x_1 : T_1, x_2 : T_2, \dots, x_n : T_n)t : R$ , or an *application*,  $t_0(t_1 : T_1, \dots, t_k : T_k)(t_{k+1} : T_{k+1}, \dots, t_n : T_n) : R$ , where  $t_0, t_1, \dots, t_n$  are terms and  $t_0$  must have a function type. A lambda abstraction can be given a name. For example, the term  $Names = \mathbf{apply}(\lambda(p)\mathbf{invoke}(name)(p))(Persons)$  is a named term returning

a set containing the names of each person in a set of Persons. **Apply** (defined in table 1), **invoke** and *name* are function symbols, *p* is a variable, and  $\lambda(p)\mathbf{invoke}(name)(p)$  is a lambda abstraction.

We note that *predicates* must be functions with *boolean* return type. Predicates are composed using AQUA's built-in operators and its term language, which is based on lambda calculus. They are normally passed as parameters to operators like **select**, **join**, **exists**, and **forall**. All queries result in the creation of a new AQUA object as the result. For example, the query discussed in the previous paragraph would result in a *new* set containing the names of each person in a set of Persons.

Some of the AQUA operations are parameterized by an equality, a type, a function, a name, or some combination of these. In the next subsections we describe type parameters and equalities, before actually describing the operations in Section 5.

We adopt some conventions and notations for defining the operators. *A* and *B* are used to refer to the input sets or multisets; *R* is used to denote the output set/multiset or the result set/multiset; *a* is used to represent an element of the input set or multiset *A*; *f*, *g* and *h* are used to represent functions; *id* represents the default equality (identity); and *p* represents a predicate. *T* indicates the result type of an operator. Tuples are represented by  $\langle \rangle$ , *L* is a tuple field name, and *a/L* means the tuple value *a* minus the field labeled *L*. Other notations will be defined as needed.

## 4.2 Type Parameters

The parameterized type constructors, and subtyping requirements for types, are designed to support static type-checking. One choice that was made to assist in this support is to explicitly give a result type as a parameter to some of the algebraic operations.

Many of the operations in our algebra construct instances of new types as their result. In such cases, inferring the result type is not easy in an algebra that allows multiple supertypes and union types. In order to resolve this difficulty and provide flexibility, the algebra takes the result type as an input parameter for operators in which we may not always have a unique supertype when combining inputs of compatible (but not identical) types. For example, consider the union of a set of oranges and a set of lemons. The type of the result can be either a set of "fruits" or a set of "good sources of Vitamin C" (Figure 1). To resolve this, the user has to specify the type of the result.

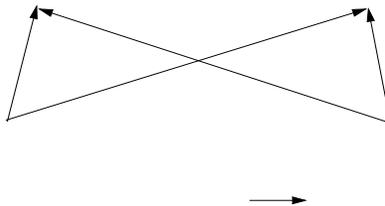


Figure 1: Result type for **union** with multiple supertypes

### 4.3 Equality

Some types may have more than one useful notion of equality. The default equality for every type is identity. The built-in primitive types (*integer*, *float*, *boolean* and *string*) have the standard definitions for equality. Meaningful user-defined equalities should induce an equivalence relation over all the instances of that type.

Equality is essential to the definition of some operators, including **union**, **intersection**, **difference**, **membership**, **LFP** (least fixed point), and **nest** for sets, and **union**, **additive\_union**, **intersection** and **difference** for multisets. These operators take a user defined equality as parameter. This equality is used to eliminate duplicates in the result.

The set **union** and **LFP** operations display the following useful property. If the result of, say, a **union** operator is used as input to another operator, then it is not necessary to eliminate duplicates when the **union** is computed. Instead, the subsequent operator(s) assume responsibility for eliminating duplicates. In this case identity should be supplied as the equality parameter for the **union**. In the case where **union** or **LFP** is the outermost operator, or the case where an existing set is viewed through a different (other than the one that created it) equality, the **dup\_elim** operator is used to eliminate duplicates according to a user specified equality. This operator takes an equality (in the form of a binary predicate) as a parameter, and eliminates duplicates under that equality. Any object that is a duplicate under identity is a duplicate under the user equality, but not vice versa.

**Dup\_elim** can be defined in terms of two other operators, **group** and **choose**. If *eq* is a binary predicate, and *S* is a set,

$$\text{dup\_elim}(eq)(A) = \text{apply}(\text{choose} \circ \text{snd}) \\ (\text{group}(\lambda(a)\text{Immutable}(\text{select}(\lambda(b)eq(a, b))(A))(A)))$$

The **Group**(*f*)(*S*) operator groups the elements of set *S* into equivalence classes by using the *f* parameter, returning a set of tuples whose first field is the result of applying *f* to a member of an equivalence class and whose second field is the set of instances in the equivalence class. In the definition of **dup\_elim**, this produces a set of tuples with two field, each of which is a set of elements which is equivalent to an element *a* of *A*. This set of tuples is supposed to represent the set of equivalence classes induced by the equality *eq* to **dup\_elim**. There is a small problem. The result of a **select** is always a new set object. This means that there will be tuples which have first elements which are not identical, but would be the same values. The second elements of such tuples would be immutable objects (values) according to the definition of **group**. Such tuples should be considered duplicates and one should be removed, but the non-identical first fields prevent the elimination of the tuples. **Immutable**(*x*) is a semantics cast, which forces the expression *x* to have immutable (value) semantics. Applying **Immutable** to the result of the **select** forces the set values to be created. Now **group** will be able to eliminate duplicate tuples representing a single equivalence class. To obtain the final result, we pick one element from each equivalence class to serve as a representative for that class. This is accomplished by extracting the second field of each tuple (using the *snd*

Definitions	
<b>apply</b> ( $f$ )( $A$ )	$= \{f(a)   a \in A\}$
<b>select</b> ( $p$ )( $A$ )	$= \{a   a \in A, p(a)\}$
<b>exists</b> ( $p$ )( $A$ )	$= \exists a \in A. p(a)$
<b>forall</b> ( $p$ )( $A$ )	$= \forall a \in A. p(a)$
<b>mem</b> ( $a, eq$ )( $A$ )	$= \exists x \in A. eq(a, x)$
<b>fold</b> ( $u, f, \oplus$ )( $A$ )	$= \begin{cases} u, & A = \emptyset \\ \bigoplus_{a \in A} f(a), & A \neq \emptyset \end{cases}$

Table 1: Unary Set Iterators

function), and the using the **Choose** operator to nondeterministically select on element of the resulting set. The **dup\_elim** set has no duplicates with respect to the new equality, thus it also has no duplicates according to identity. See Tables 1 and 4 for definitions of **apply**, **choose**, **group**, and **select**.

For many queries, this approach allows any equality to be used at any point during query processing without compromising our notion of “set”. The operators remain defined in the abstract and we are assured that they can handle any kind of equality that may arise, including anything the writer of a query might wish to pass in as a parameter.

Consider the union of two sets  $A = \{(1, a), (2, b)\}$  and  $B = \{(2, a), (2, b)\}$ . Assume we want  $A$  union  $B$  using a notion of equality that says two elements are equal if their fields are pairwise equal, so that the  $(2, b)$  in  $A$  and the  $(2, b)$  in  $B$  are equal. The result will then have three elements:  $\{(1, a), (2, b), (2, a)\}$ .

Now suppose we want  $A$  union  $B$  using a notion of equality that says two elements are equal if their second fields are equal. The  $(1, a)$  in  $A$  and the  $(2, a)$  in  $B$  are also equal. The required result is then  $\{(1, a), (2, b)\}$  or  $\{(2, a), (2, b)\}$ . Either result is correct since the equality only examines the second element of each tuple.  $(5, a)$  could legally be in the result, but is disallowed by our definition of **dup\_elim** – it was not in the set before **dup\_elim** was applied.

## 5 The Operators

In this section we describe the operations defined on the different types. Some of these operators can be expressed in terms of some others, leading to many redundancies in the operator set. They have been retained partly because they permit some expressions to be written with greater conciseness and clarity than would otherwise be possible, and partly because they lend themselves to specialized implementations and optimizations that can be more efficient than those of a more general operator.

### 5.1 Set Operators

This subsection describes the set operators in our algebra. Most of these operators are derived from similar operators in the literature [9, 28, 32] and exceptions are noted as they arise. It is our combination and utilization of them, in

Definitions	
$\mathbf{union}(eq, T)(A, B)$	$= \mathbf{dup\_elim}(eq)(\{x \mid (x \in A \text{ or } x \in B)\})$
$\mathbf{intersect}(eq, T)(A, B)$	$= \mathbf{dup\_elim}(eq)(\{x \mid (x \in A \text{ or } x \in B) \text{ and } (\mathbf{mem}(x, eq)(A) \text{ and } \mathbf{mem}(x, eq)(B))\})$
$\mathbf{diff}(eq, T)(A, B)$	$= \mathbf{dup\_elim}(eq)(\{x \mid x \in A \text{ and } \neg(\mathbf{mem}(x, eq)(B))\})$

Table 2: Binary Set Operators

Definition	
$\mathbf{LFP}(eq, f)$	$= \mathbf{dup\_elim}(eq)(\bigcup_{i=0}^{\infty} (f^i(\{\}))$

Table 3: Least Fixed Point Operator

Definitions	
$\mathbf{set}(a)$	$= \{a\}$
$\mathbf{choose}(A)$	$= \text{some } a \in A$
$\mathbf{group}(f)(A)$	$= \{(f(a), eqclass(a)) \mid a \in A\},$ where $eqclass(a) = \{a' \mid a' \in A, f(a) = f(a')\}$
$\mathbf{dup\_elim}(eq)(A)$	$= R \subseteq A \text{ s.t. } \forall x, y \in R, eq(x, y) \Rightarrow id(x, y),$ and $\forall x \in A, \exists y \in R \text{ s.t. } eq(x, y)$
$\mathbf{nest}(eq, L)(A)$	$= \{\mathbf{tup\_concat}(a/L, <L : \{b.L \mid b \in A \text{ and } eq_i(a/L, b/L)\} >) \mid a \in A\}$
$\mathbf{unnest}(L)(A)$	$= \{\mathbf{tup\_concat}(a/L, <L : s >) \mid a \in A \text{ and } s \in a.L\}$
$\mathbf{convert}(A)$	$= \{*x \mid x \in A*\}$

Table 4: Set Restructuring Operators

Definitions	
$\mathbf{join}(p, f)(A, B)$	$= \{f(a, b) \mid a \in A, b \in B, p(a, b)\}$
$\mathbf{tup\_join}(p)(A, B)$	$= \mathbf{join}(p, \mathbf{tup\_concat})(A, B)$
$\mathbf{outer\_join}(p, f, g, h, T)(A, B)$	$= \{f(a, b) \mid a \in A, b \in B, p(a, b)\} \cup \{g(a) \mid a \in A, \forall b \in B, \neg p(a, b)\} \cup \{h(b) \mid b \in B, \forall a \in A, \neg p(a, b)\}$

Table 5: Join Operators

addition to the original operators, that makes this approach more flexible than previous ones.

A list of all the operators for sets and a brief definition for each of them, is given in tables 1 through 5. Table 1 lists the unary set operators, table 2 deals with the binary set operators, table 3 defines the least fixed point (**LFP**) operator, table 4 describes the set restructuring operators, and table 5 lists the various join operators. In the rest of the subsection, we expand upon issues about some of the operators of the algebra.

The **fold** operator is a powerful operator –  $\mathbf{fold}(u, f, \oplus)(A)$  reduces set  $A$  to a single value by applying  $f$  to each element and iteratively combining the results with a dyadic operator  $\oplus$ .  $u$  is the result of **fold** on the empty set. For example, **set\_collapse** can be implemented using **fold**, the identity function, and the **union** operator (using the default equality).

$\mathbf{fold}(\{\}, \lambda(x)x, \mathbf{union}(id))(\{\{1, 2\}, \{2, 3, 4\}, \{5\}\}) = \{1, 2, 3, 4, 5\}$

Operators **exists**, **forall** and **mem** return a *boolean* value and can be used as predicate formers. **Nest** and **unnest** have been defined in table 4 using a single tuple field name  $L$ . However, this definition can be easily extended to a list of field names. In such a case,  $a/L$  refers to the tuple value  $a$  minus the fields in the list  $L$  and  $a.L$  is the concatenation of all the values of the fields in list  $L$ . To compare the equality of a pair of tuples minus the field  $L$ , we use the equality  $eq_t$ , which does a pairwise comparison of the contents of each field using the equality  $eq$ .

The binary set operators, **union**, **intersection** and **difference** are the familiar set-theoretic operations; however our definitions are complicated by considerations of typing. When two sets are combined using a binary set operator, it is not necessary that they have the same type. As in EXCESS [9], these operators take an extra argument that specifies the type of the result, as discussed in subsection 4.2. The result type of **union** has to be a supertype of the types of the input sets. However, in **intersection** the result type can either be a supertype of both input types or be one of the input types. In the case of **difference**, the result type has to either be the type of the first input set or one of its supertypes.

To briefly illustrate some of the set operators, consider a query that finds all of the people who live in the same city in which they work and groups them based on the name of this city. This is done by using the employer field of a *Person* object. We use  $A.B$  as a shorthand for **invoke** ( $A, B$ ), which invokes method  $B$  on object  $A$ .

$LiveWhereWorkPeople =$   
 $\mathbf{select}(\lambda(x)x.address = x.employer.address)(Persons)$

Next, we use **group** to group the people in *LiveWhereWorkPeople* by the city in which they live.

$\mathbf{group}(\lambda(x)x.address)(LiveWhereWorkPeople)$

The result consists of a set of ordered pairs (*city, people*) where *city* is the name of a city in which at least one person both lives and works and *people* is

Definitions	
<b>multiset</b> ( $a$ )	$\{ * a * \}$
<b>convert</b> ( $A$ )	$\{ x   x \in A \}$
<b>dup_elim</b> ( $eq$ )( $A$ )	<b>convert</b> ( <b>dup_elim</b> ( $eq$ )( <b>convert</b> ( $A$ )))

Table 6: Multiset Restructuring Operators

a set of *Person* objects all of whom live and work in *city*.

The various **join** operators deserve special mention due to their generality. **Join** takes a function as a parameter, thus allowing the user to define a “combining” function. The other **join** operators are similar generalizations involving a predicate and a function. Note that the union used in the definition of **outer\_join** is the set **union**, using the default equality. The resultant type  $T$  of the **outer\_join** must be a supertype of the result types of functions  $f$ ,  $g$  and  $h$ , to allow unioning the results of the functions. Left and right outer joins can be expressed in terms of **outer\_join** with the appropriate interpretation of *null* values. Familiar join operators like **natural\_join**, **equijoin**, **semijoin** and **antijoin** are not primitives in the algebra, but they can be expressed easily in terms of the included join operators.

The **LFP** operator [17] is defined in table 3. The function  $f$  is of type  $T \rightarrow T$ , where  $T$  is a set type. The notation  $f^i$  is equivalent to  $f(f^{i-1})$ . The **union** for LFP uses the equality parameter for **LFP** as its equality parameter, and its type parameter,  $T$ , must be the input/result type of  $f$ . We also make the restriction that  $f$  be “monotonic” in the sense that  $A \subseteq B \Rightarrow f(A) \subseteq f(B)$ .

## 5.2 Multiset Operators

Multisets support nearly all the same operations as sets, with very similar semantics in most cases. The difference between a multiset and a set is that a multiset may contain multiple occurrences of the same element. The notation used to denote multisets is  $\{ * e_1, e_2, \dots, e_n * \}$ , where  $e_i$  are the elements of the multiset. A multiset former is similar to a set former, so the multiset former  $\{ * x | x \in M \text{ and } x > 1 * \}$  is read as “the multiset of items  $x$ , such that each occurrence of  $x$  in  $M$  is greater than one”. We define the *cardinality of an element* of a multiset as the number of occurrences of that element in the multiset. The notation  $|A|_a$  means “the cardinality of  $a$  in multiset  $A$ ”. Similarly, the notation  $|A|_{eq(a)}$  means “the cardinality of elements “equal” to  $a$  (where equality is determined by the equality operator,  $eq$ ) in multiset  $A$ ”. We will also speak of the *cardinality of a multiset*  $|A|$ , meaning its total element count, tallying duplicates as many times as they occur. *Multiset*[ $T$ ] denotes a multiset containing objects of type  $T$ .

Most multiset operators are quite similar to the corresponding set operators, except for the fact that the input and output types are multisets instead of sets. Most of the formal definitions in tables 1, 3, 4, and 5 hold for multisets too.

The exceptions are **multiset**, **dup\_elim**, **convert**, defined in table 6; and **union**, **additive\_union**, **intersection** and **difference** which are defined in table 7.

In the binary operators on multisets, we find the greatest departure from the corresponding set operators. All the binary operators are based on the

Definitions	
$\mathbf{union}(eq, T)(A, B)$	$=$ $R : \text{Multiset}[T]$ such that $\forall x : T$ , $ R _{eq(x)} = \max( A _{eq(x)},  B _{eq(x)})$ Also, $x \in R \Rightarrow (x \in A) \text{ or } (x \in B)$
$\mathbf{additive\_union}(eq, T)(A, B)$	$=$ $R : \text{Multiset}[T]$ such that $\forall x : T$ , $ R _{eq(x)} =  A _{eq(x)} +  B _{eq(x)}$ Also, $x \in R \Rightarrow (x \in A) \text{ or } (x \in B)$
$\mathbf{intersect}(eq, T)(A, B)$	$=$ $R : \text{Multiset}[T]$ such that $\forall x : T$ , $ R _{eq(x)} = \min( A _{eq(x)},  B _{eq(x)})$ Also, $x \in R \Rightarrow (x \in A) \text{ or } (x \in B)$
$\mathbf{diff}(eq, T)(A, B)$	$=$ $R : \text{Multiset}[T]$ such that $\forall x : T$ , $ R _{eq(x)} = \max(0,  A _{eq(x)} -  B _{eq(x)})$ Also, $x \in R \Rightarrow (x \in A) \text{ or } (x \in B)$

Table 7: Binary Multiset Operators

cardinality of the elements in the two input sets (table 7). For example, **union** in a multiset is,

$$\mathbf{union}(id, Int)(\{ * 1, 1, 2 * \}, \{ * 1, 2, 2 * \}) = \{ * 1, 1, 2, 2 * \}$$

However, regarding the typing of the arguments and the result, binary multiset operators are similar to the set operators. We also define **additive\_union** for multisets.

### 5.3 Other Type Operators

Besides sets and multisets, the algebra supports a host of other types. The union type along with its constructor allows creation of discriminated unions. The operations defined for the type are **union**, **tagcase** and **typecase**. **Union**( $U, tag, e$ ) creates an instance of union type  $U$  and initializes its contents to be entity  $e$  with tag  $tag$ . Both **tagcase**( $e$ ) and **typecase**( $e$ ) selectively execute a set of terms based either on the tag or the type of the union instance  $e$ .

Function types represent functions, which take some number of typed parameters and return a single typed result. Instances of function types are created by the use of typed lambda expressions. These instances cannot be tested for equality, since it is impossible to compare two functions.

Tuples are records with named fields, with the familiar operators for instance creation (**tuple**), concatenation (**tup\_concat**), and field selection (**select\_field** or infix “.”). Sufficient care is taken to avoid duplicate field names in **tup\_concat**.

Boolean is actually a type rather than a constructor. Booleans are used to represent truth values for conditionals and are provided as the result of comparisons and quantifiers (the set operators **exists** and **forall**). Operations on booleans are **and**, **or**, and **not**.

Abstract data types are composite types whose elements are accessed only via a set of functions, which are called the interface. The functions are accessed via the **invoke**( $I, f$ ) operator which invokes  $f$  on instance  $I$ .

Type	Operators
<b>Set</b>	Set, Choose, Union, Fold, LFP
<b>Multiset</b>	Multiset, Choose, Additive_Union, Fold, LFP
<b>Union</b>	Make_union, Tagcase, Typecase
<b>Tuple</b>	Tuple, Select_field
<b>Boolean</b>	And, Not
<b>Abs</b>	New, Invoke

Table 8: Primitive AQUA Operators

## 5.4 Primitive Operators

Table 8 provides a minimal set of *primitive* operators<sup>1</sup> for each type. All non-primitive operators can be defined in terms of the primitive operators for the type. For example, **mem** for a set can be defined in terms of **fold**:

$$\mathbf{mem}(eq, a)(A) = \mathbf{fold}(false, \lambda(x)eq(x, a), \mathbf{or})(A)$$

Similarly, **select** can also be expressed in terms of **fold**:

$$\mathbf{select}(p)(A) = \mathbf{fold}(\{ \}, \lambda(x)\text{if } p(x) \text{ then } \mathbf{set}(x) \text{ else } \{ \}, \mathbf{union}(id, T))(A)$$

**Dup\_elim** can also be expressed using **fold** with **set** and **union** (using identity as the union equality). Indeed, most operations (e.g., **union**, **intersect** and **diff** with equality parameters, **apply**, **group**) can be defined with those three primitive operations.

## 6 Conclusions

This paper has briefly summarized the AQUA data model and algebra. It is proposed as the input language for object-oriented query optimizers. It has been designed to cover the functionality of many existing query languages, and to provide the maximum potential for optimization. As a result, the set of operators is purposefully not minimal. We have illustrated its use with a few simple examples.

The AQUA data model embodies a uniform approach to objects and values. Values are simply immutable objects. They are objects in all other respects. They have an abstract interface, and they possess an identity that can be used to refer to them.

A type describes syntactic properties of objects and their methods. Semantic properties of a type are supplied by an axiomatic specification, called its *semantics*, that is separated from the type definition (i.e., syntax). Immutability is an example of something that would be specified in the semantics. A given type can be associated with multiple semantics, and each of these semantics can be implemented in many ways. Currently we provide a default mechanism for determining the semantics of objects that are results of algebraic queries and we provide a mechanism for overriding this default.

<sup>1</sup>The primitive set **union** in the table uses identity for equality

This paper has discussed algebraic operators for the *Set* and the *Multiset* types. We also propose an extension to AQUA to include algebraic operators for other bulk types such as *List*, *Tree*, and *Graph* [31].

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